TRINITY COLLEGE

YEAR 12
MATHEMATICS
SPECIALIST

Test 1, 2023
Section One: Calculator Free
Complex Numbers and Functions

STUDENT'S NAME: _____

DATE: Thursday 16th March **TIME:** 40 minutes **MARKS**: 40
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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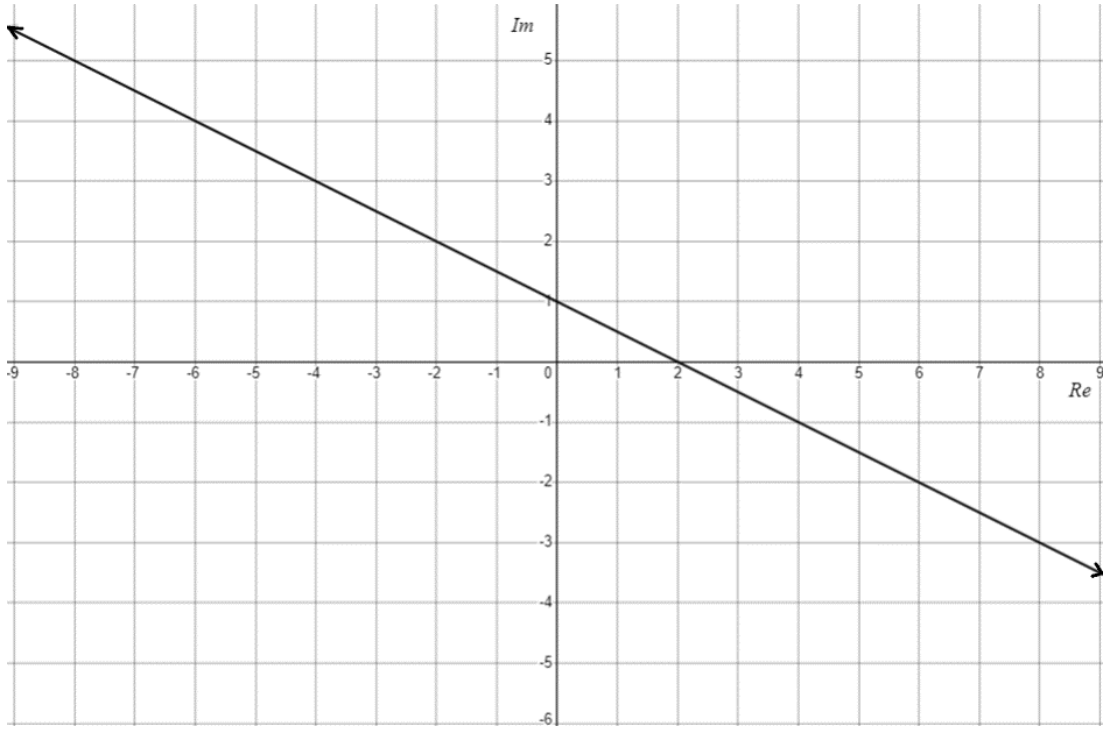
Question 1 **(10 marks)**

Consider the following complex locus for z.



(a) Represent the above locus using, but not necessarily limited to, the following.

i) Using the Absolute Value/Magnitude function. (2 marks)

ii) Using $Re(z)$ and/or $Im(z)$. (1 mark)

iii) Using $Arg(z + a)$, $Arg(z + b)$ and ∪ (union), where $a, b \in \mathbb{C}$. (2 marks)

Consider the following locus $|w - 6 - 2i| = 2$.

(b) Sketch the locus on the diagram provided at the beginning of this question. (2 marks)

(c) Determine the maximum and minimum values of $Arg(w)$.
Include a sketch in your working out to aid your response. (3 marks)

Question 2 **(3 marks)**

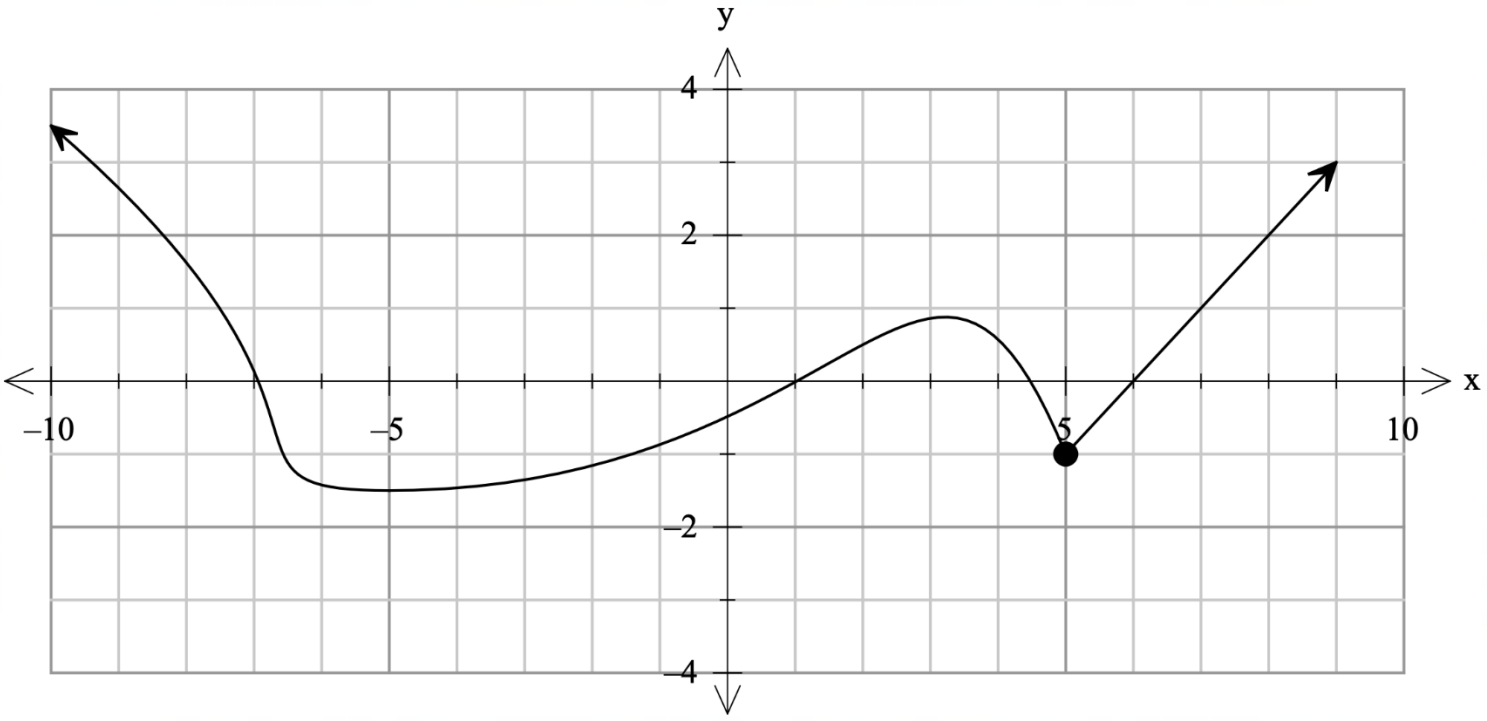
Prove and state the conditions of $|z|$ and $Arg(z)$ for which $\bar{z} = z^{-1}$, $z \in \mathbb{C}$.

Question 3 **(5 marks)**

Consider the function $f(z) = z^4 - 4z^3 + 9z^2 - 16z + 20$ where $z \in \mathbb{C}$.
Solve $f(z) = 0$ if $z - (2 + i)$ is a factor of $f(z)$.

Question 4 **(9 marks)**

Consider the function $f(x)$ which has been sketched on the provided axis.



(a) On the above axis sketch $f(|x|)$ over the domain $x \leq 0$. (2 marks)

(b) On the above axis sketch $\frac{1}{f(x)}$ over the domain $x \geq 0$. (3 marks)

(c) State when $f(x) = |f(x)|$. (1 mark)

(d) $g(x) = f(x)$ for the values $5 \leq x \leq 6$.
If $g(x) = -|ax + b| + c$, determine the values of a, b and c. (3 marks)

Question 5 **(3 marks)**

Calculate and state the nature of all asymptotes to the equation $f(x) = \frac{x^2+4x-7}{x-1}$.

Question 6 **(10 marks)**

Consider the two equations: $f(x) = \sqrt{x-9}$ and $g(x) = -ax^2 + c \quad a, c \in \mathbb{R}, \quad a \geq 0, \ c \geq 9.$

(a) Determine $f^{-1}(x)$ and state the domain and range. (3 marks)

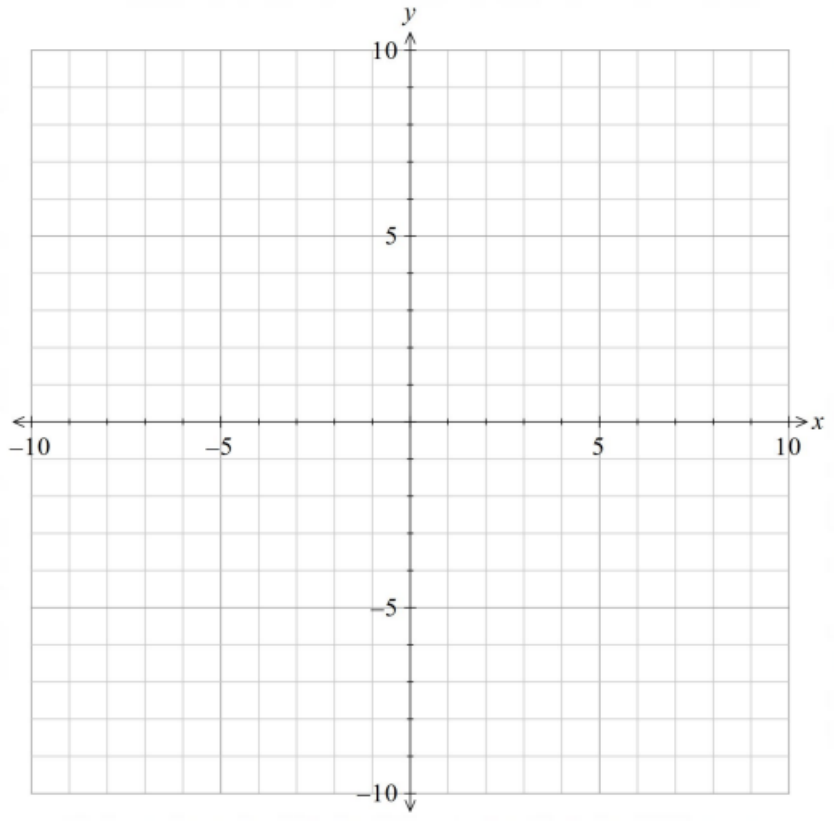
(b) Determine $f(g(x))$. (1 mark)

(c) Show and briefly explain how the generalised range of $f(g(x))$ is $0 \leq y \leq \sqrt{c-9}$. (2 marks)

(d) Show and briefly explain how the generalised domain of $f(g(x))$ is (2 marks)

$$\frac{-\sqrt{4a(c-9)}}{2a} \leq x \leq \frac{\sqrt{4a(c-9)}}{2a}$$

(e) Sketch $f(g(x))$ if $a = 1$ and $c = 25$. (2 marks)



END OF QUESTIONS

TRINITY COLLEGE

YEAR 12
MATHEMATICS
SPECIALIST

Test 1, 2023
Section Two: Calculator Allowed
Complex Numbers and Functions

STUDENT'S NAME: _____

DATE: Thursday 16th March **TIME:** 10 minutes **MARKS**: 10
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Question 7 **(10 marks)**

Consider the equation $z^5 = 16 + 16\sqrt{3}i$, where $z \in \mathbb{C}$.

(a) Give exact solutions to the equation in the form r$cis\theta$ where $0 \leq \theta \leq 2\pi$. (4 marks)

If w is the solution with the smallest argument and u is the solution with the largest argument:

(b) State w and u. (1 mark)

(c) Determine $Re(wu)$. (2 marks)

(d) Determine $|3w^3|$. (1 mark)

(e) Determine a if $a \in \mathbb{R}$ and $Arg(w + a) = \frac{\pi}{2}$. (2 marks)

END OF QUESTIONS